

Algebra is the fundamental language of mathematics.

It is more than the subject
taught in 7th and 8th grade...

Algebraic thinking begins in our
Pre-kindergarten classes.

The
Enhancing Mathematical Learning
Class
2003-2004

presents

Algebra Through the Grades

Dedham Public Schools

Dedham, Massachusetts

Part One

- NCTM Algebra Standard
- Instructional Goals
- Essential Aspects of Algebraic Thinking

*NCTM Principles and Standards for
School Mathematics*

Algebra Standard

"By viewing algebra as a strand in the curriculum from pre-kindergarten on, teachers can help students build a solid foundation of understanding and experience as a preparation for more-sophisticated work in algebra in the middle grades and high school."

Instructional Goals for Algebraic Thinking

- build students' understanding of equivalence, patterns and functions, variables, and graphing;
- link algebraic thinking instruction to arithmetic instruction;
- incorporate manipulative materials, children's books, familiar contexts, and problem-solving experiences;
- help students see arithmetic, geometry, and algebra as connected topics; .establish a classroom atmosphere that stimulates and supports students' learning.

Essential Aspects of Algebraic Thinking Include:

- Equivalence
- Students should understand the equal sign as an indication that quantities or expressions have the same value, not as a signal to write the answer.
- Patterns and Functions
- Students should learn how to recognize, create, and extend growth patterns; describe patterns verbally; and represent patterns numerically, algebraically, and geometrically.

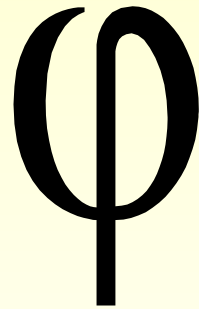
- Variables
- Students should understand the various uses of variables, including using symbols to stand for an unknown value; describe a relationship between two quantities; represent mathematical properties (e.g., $a + b = b + a$); and represent quantities in formulas.
- Graphing
- Students should learn how to use ordered pairs of numbers to plot points on a coordinate grid for the purpose of representing relationships between two quantities.

Adapted from Lessons for Algebraic Thinking (Grades K-2, 3-5, 6-8) Math Solutions Publications Booth 116 by Marilyn Burns

Part Two

Algebra in Grades 1-7

Algebra in Grade 1

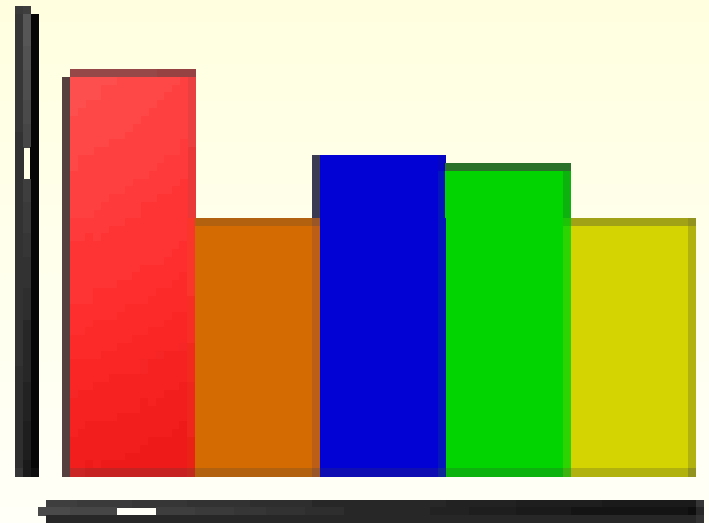


By:

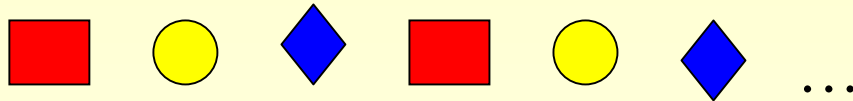
Clare Sullivan, Michelle Joyce, Barbara Cross, Cheryle Biggar, and Heidi Krug

Algebra in Grade 1

- In first grade students explore pre-algebraic and algebraic thinking.
- Students are exposed to the concepts of variables, patterns, equality, and representation.
- This exploration includes the use of manipulatives, mental math, and problem solving.



Patterns & Rules



Pattern Questions...

- ABCABCABC...

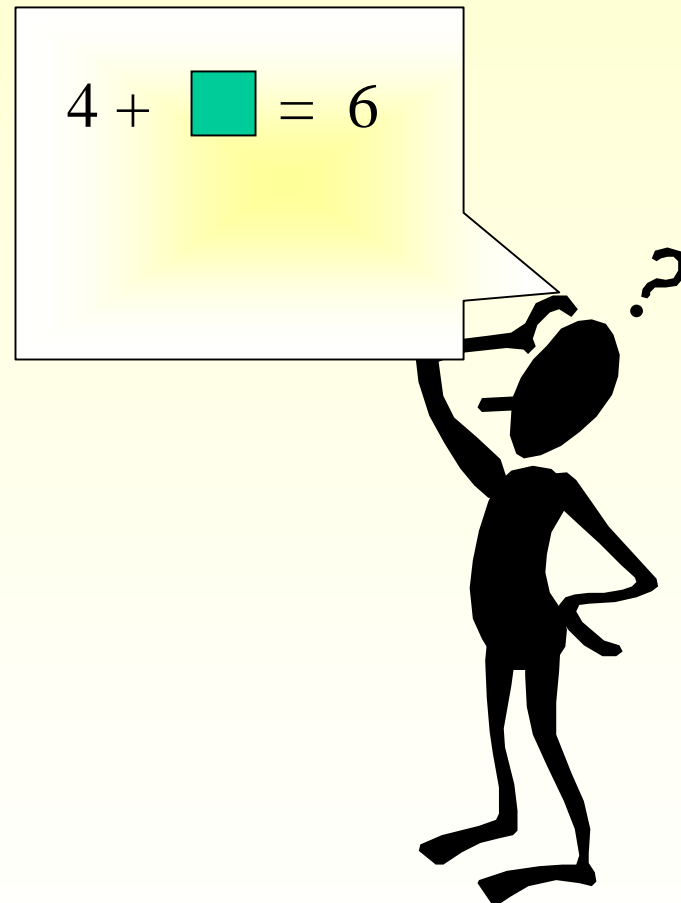
- 1 2 3 1 2 3...

- 5, 10, 15, 20

1. What is the pattern?
2. What would the 10th _____ be?
3. How else could you show this pattern?
4. Make a new pattern.

Variables

- Students solve for the unknown variable in addition and subtraction number sentences.
- Later, in word problems, they must determine the operation to find the unknown.



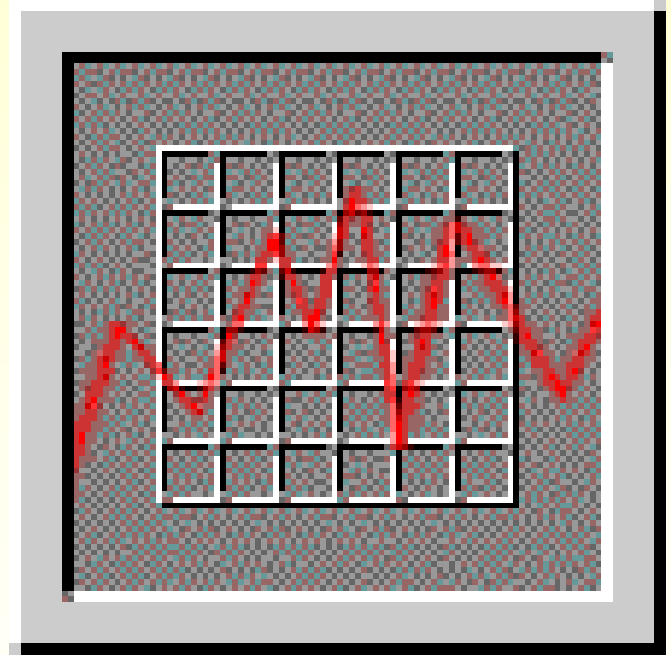
Equality

- First graders learn that you can add numbers in any order to get the sum. (Commutative Principle)
- When subtracting, students learn you have to start with the greater number.
- Students learn to write, solve and create number sentences.
- The students learn that the = is like a balance scale. Both sides must be the same amount.

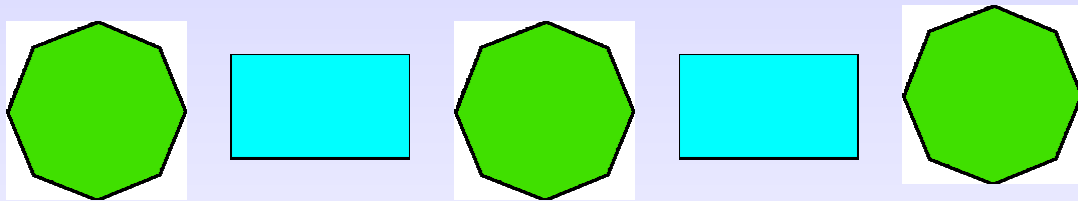


Representation

- In first grade children learn to read, interpret, and create graphs.
- These include picture, bar, and circle graphs.
- Representing numbers through the use of manipulatives is vital to the concept of place value.



Algebra in Grade 2



By:

Clare Sullivan, Michelle Joyce, Heidi Krug,
Barbara Cross, Cheryle Biggar

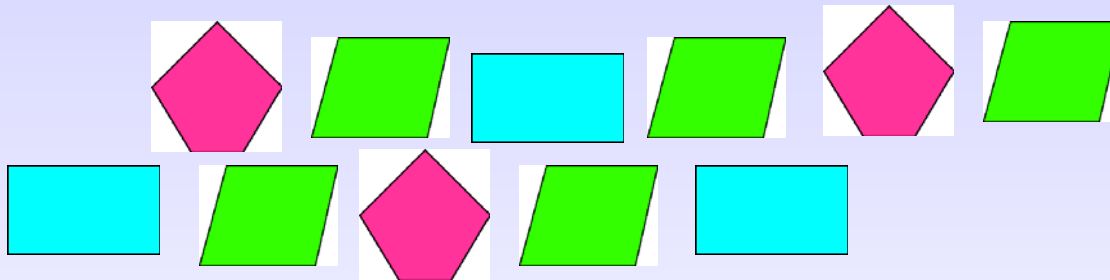
Algebra in Grade 2

- ❖ In second grade students continue to explore pre-algebraic and algebraic thinking.
- ❖ Students continue their exposure and work with variables, patterns and rules, equality and representation.
- ❖ The exploration continues to use manipulatives, mental math and problem solving to extend the skills introduced in first grade.



Patterns

Pattern activities begin with shapes.



- What is the pattern?
- Can you represent the pattern in another way(using numbers, letters, etc.)?
- What would be the shape of the 20th piece?
- How many pentagons will be needed to complete the pattern for 30 pieces?

Patterns

Students in second grade continue to work with patterns involving shapes. They move on to patterns with numbers- skip counting and arithmetic sequences.

Skip counting- 2, 4, 6, 8, _____, _____

5, 10, 15, 20

Arithmetic sequences- 1, 5, 9, _____, _____

(+4)

Variables

Missing Addends and Subtrahends

Students learn to solve for the missing number through facts to 20.

$$9 + \square = 18$$

$$15 - \square = 7$$

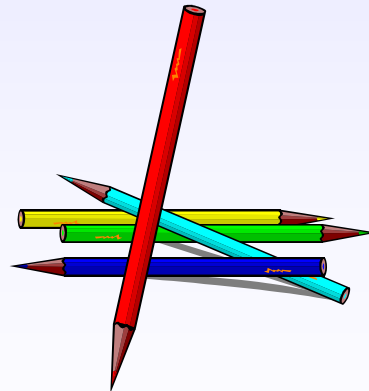


Variables

Students move from solving simple equations with missing numbers to problem solving.

Mary had 4 pencils. She bought some more pencils. Now she has 8 pencils. How many pencils did Mary buy?

$$4 + \square = 8$$



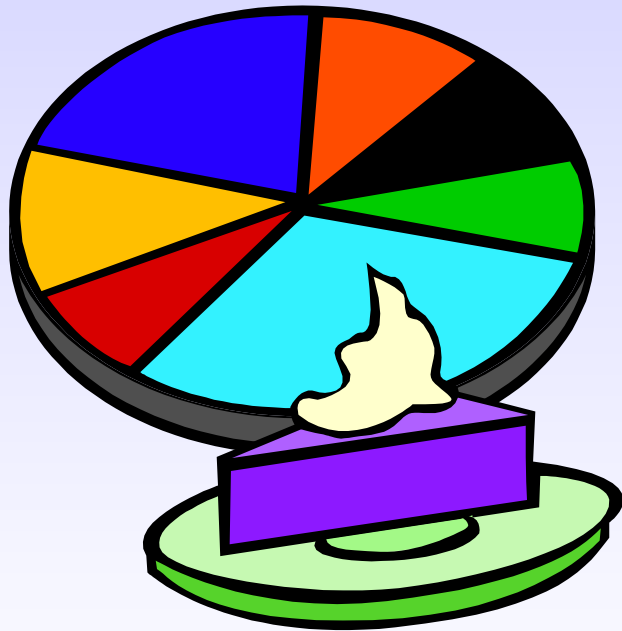
Equality



- $6 + 4 = 4 + \square$
- $7 = 9 - \square$

Students are taught that equality is like a level balance scale. Equations on each side of the sign must be equal.

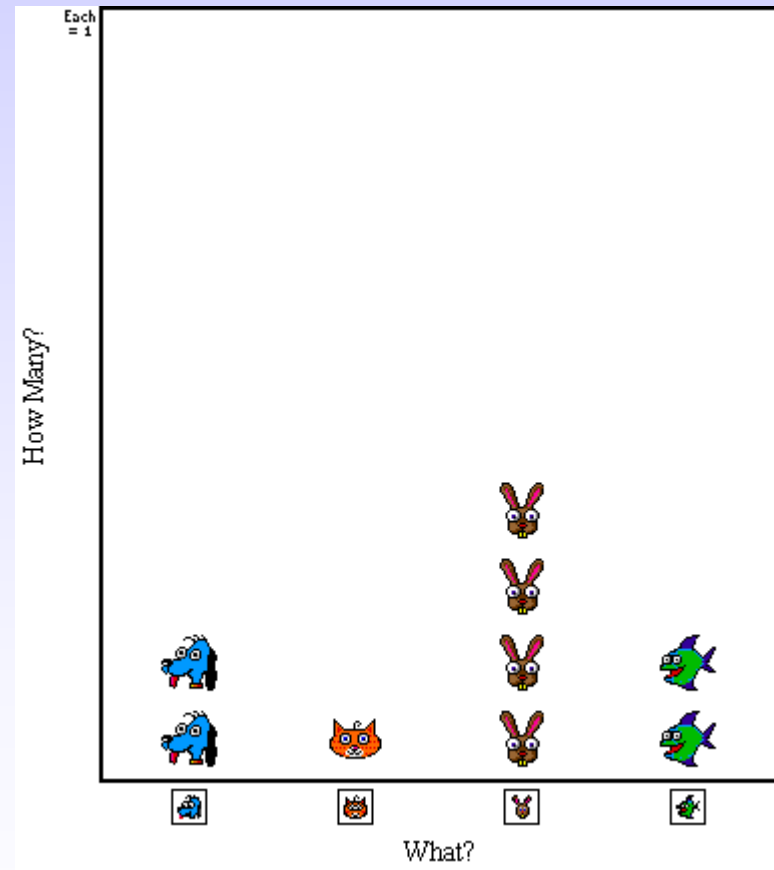
Representation



- Children learn to display mathematical relationships graphically, symbolically, pictorially, or verbally.

Representation

Students collect data and learn how to create graphs to show the results.



Algebra in Grade 3



By: Cheryl Fish, Karen Giannangelo, Heidi Dineen, Joan Donovan

Algebra in Grade 3

Strand: Patterns, Relations, and Algebra

4.P.1 (Gr.3) Create, describe, extend, and explain symbolic (geometric) and numeric patterns, including multiplication patterns up to 1,000.

18, 27, 36, 45, _____, _____

Algebra in Grade 3

4.P.2 (Gr. 3) use symbol and letter variables

(e.g. \square , x) to represent unknowns or qualities that vary in expressions and in equalities (mathematical sentences that use $=$, $<$, $>$, $)$ (up to 1,000)

Use $>$, $<$, or $=$

$$21 \times 2 \quad \square \quad 21 \times 0$$

$$4 \times 7 \quad \square \quad 7 \times 4$$

Algebra in Grade 3

4.P.3 (Gr.3) Determine values of variables in simple equations.

$$54 - \text{circle} = 22$$

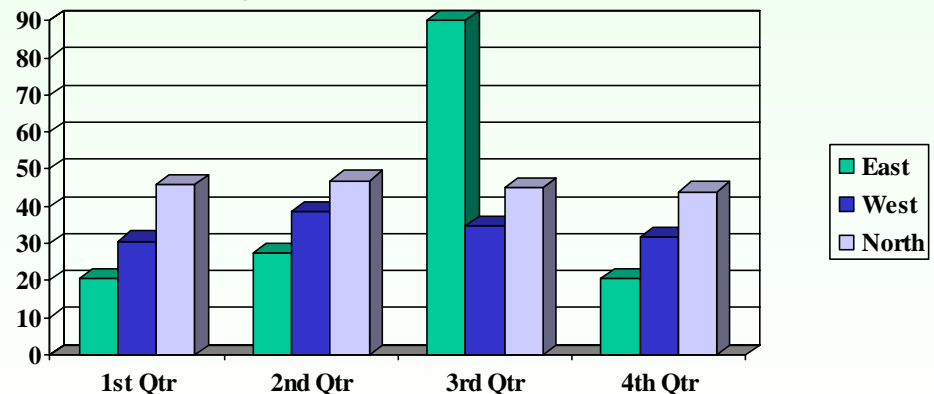
$$16 = \text{triangle} + 8$$

$$\text{diamond} + \text{square} = 13$$

Algebra in Grade 3

4.P.4 (Gr. 3) Use pictures, models, tables, charts, graphs, picture bar, number sentences, and mathematical notations to interpret mathematical relationships.

Which direction is traveled most during the four quarters of the school year?



Algebra in Grade 3

4.P.5. (Gr. 3) Introduce solving problems which involve simple proportional relationships, including unit pricing.

# of Gumballs	Cost
1	
3	30 cents
5	50 cents
7	

Algebra in Grade 3

4P.6. (Gr. 3) Determine how change in one variable relates to a change in a second variable.

In (n)	3	4	5	6
Out (n+1)	4	5	6	7

Algebra in Grade 3

Input	Output
10	20
20	30
5	15
17	27
	45
25	
0	
	39
N	

Mystery Machine

- What does the mystery machine do?
- If you are given an output number, how can you find the input number?

Algebra in Grade 4



By
Jean Weir

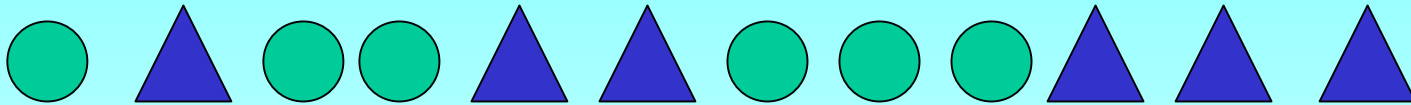
Algebra in Grade 4



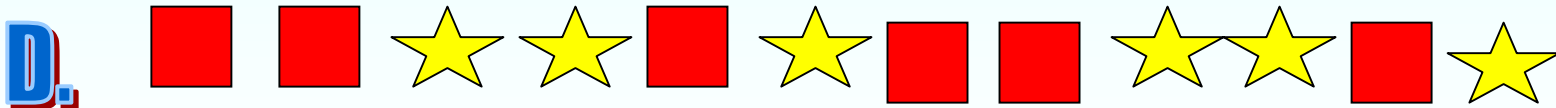
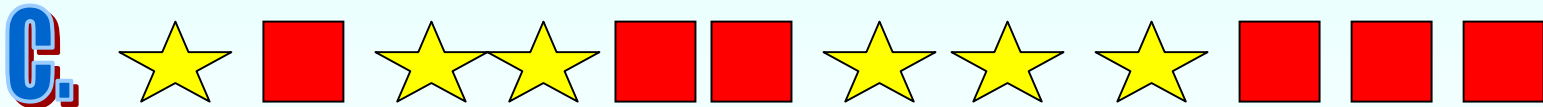
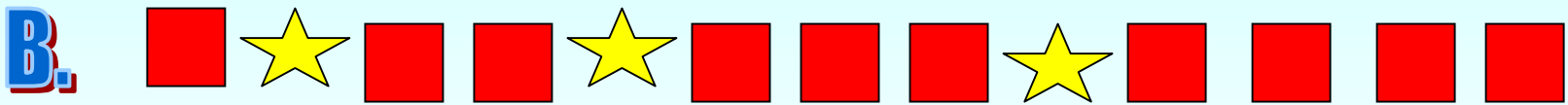
Grade four continues to develop patterns, properties and expressions that have been introduced in the primary grades. Function tables with rules stated using variables is introduced at this level. The area of algebra that is mostly introduced in Grade 4 is writing equations and inequalities.

Patterns

These shapes are arranged in a pattern.

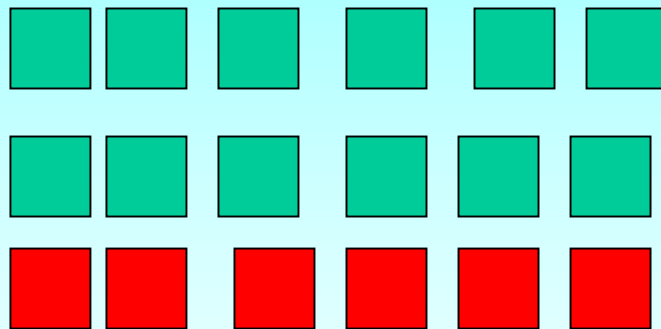


Which set of shapes is arranged in the same pattern?



Distributive Property

$$3 \times 6 = (2 \times 6) + (1 \times 6)$$



3 rows of 6 is the same as 2 rows of 6 and 1 row of 6.

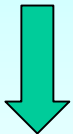
Writing/Evaluating Expressions with Variables

The train at the zoo has 6 cars. How many people does the train hold if each car holds n people?

Write an expression.

Let n = the number of people each car holds.

Number of cars



6

n

People in each car



x







The train holds $6n$ people.

Algebra in Grade 5



By
Nancy R. Nichols

Algebra in Grade 5

-  Algebraic expressions with parenthesis
-  Numerical and geometric patterns: sequences and repeating
-  Understanding variables
-  Properties of equality
-  Relationships
-  Integers

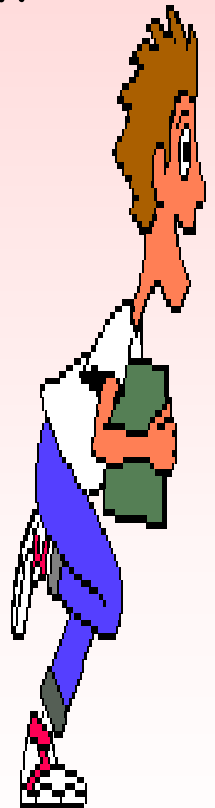


Algebraic Expression

After 8 weeks, Mia's plant was h inches tall, and Fred's plant was 2 inches taller than Mia's plant. Write an expression for the height of Fred's plant and evaluate the expression for $h = 7, 8, 9$

Expression: $h + 2$

h	$h + 2$
7	$7 + 2 = 9$
8	$8 + 2 = 10$
9	$9 + 2 = 11$



Patterns

A pattern is an arrangement of things repeated in an orderly and recognizable fashion. Numbers in a problem that are not given can be found by using the established pattern.

CRACK THE CODE!

A.) 100, 1,000, 200 _____ 300, 3,000, _____, _____, _____

B.) 10, 19, 20, 29 _____, 39, _____, _____, _____

C.) 11, 1, _____, 2, 33, _____, 44, 4 _____

D.) 123, 120, 114, _____, 93, _____, _____, 39, 15

Properties of Equality

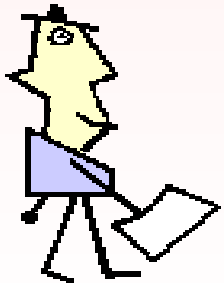
In an equation, the expressions on each side have the same value.

Addition Property: $8 + 4 = 7 + 5$ So, $8 + 4 + 3 = 7 + 5 + 3$

Subtraction Property: $9 + 15 = 24$ So, $(9 + 15) - 4 = 24 - 4$

Multiplication Property: $7 \times 6 = 14 \times 3$ So, $7 \times 6 \times 2 = 14 \times 3 \times 2$

Division Property: $12 + 8 + 20$ So, $(12 + 8) \div 5 = 20 \div 5$



Order of Operations

Parenthesis First, Exponents, Multiplication or Division, Addition or Subtraction

Please Excuse My Dear Aunt Sally.

$$(5 + 9) \times 2 \div 4 =$$

$$(3 + 2) \times 8 \div 8 =$$

Students use mnemonics to remember their order of operations.

Algebra in Grade 6

Part I



By
Amy Gilbert

Patterns and Algebraic Expressions

In Grade 6, "Algebraic Thinking" is first introduced in the very first lesson of the *Mathematics* textbook. The first section of the Module 1 is entitled "Patterns and Sequences". They are given experiences with both arithmetic and geometric sequences.

Patterns and Algebraic Expressions

Examples:

Arithmetic Sequences:

1, 3, 5, 7, 9,...

XY, XXYY, XXXYYY, XXXXYYYY,...

00, 0000, 000000, 00000000,...

Geometric Sequences:

6, 12, 24, 48, 96,...

100, 50, 25, 12.5,...

4, 20, 100, 500, ...

Patterns and Algebraic Expressions

General Rules

After identifying, describing, extending, and creating repeated patterns using pattern blocks, symbols, numbers, and number words, students learn how the key terms "term numbers" and "term" are related.

Students learn to create "General Rules" by relating a term number in a sequence to its term.

Term #	1	2	3	4	5
Term	5	10	15		

Students have to think: "What is done to the term number to generate its term?" In this example, they would have to recognize that the term number is multiplied by 5 to get the term.

Soooo... If asked, "What is the 20th term?", students think: "20 X 5 = 100." and say: "Multiply 20 by 5 to get the 20th term. The 20th term is 100." In a later section, students learn to write equations (with variables) to generate the nth term of a sequence.

Term #	1	2	3	4	5
Term	5	10	15		

Algebra in Grade 6

Part II



By

Lauren O'Connell

Game - "Guess My Rule"

- Students work in pairs
- Student one is given a card with a rule on it, such as: multiply the input by 7.
- Student two says a number.
- Student one applies the rule to student 2's number and then gives the answer.
- Student two then has to guess what the rule is

Why "Guess My Rule?"

- Students become exposed to solving algebraic equations in a fun, non-threatening way.



Sequence Tables For Multiples

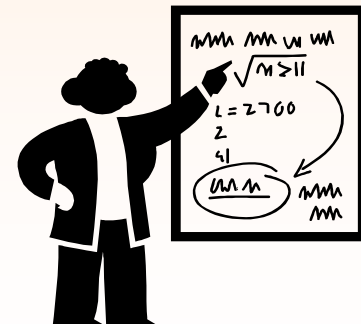
- Next, students reinforce the concept of general rules, using multiples.

Multiples of 5:

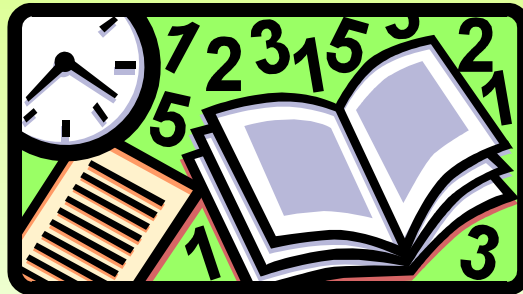
Term #	1	2	3	4	5	n
Term	5	10	15	20		

Why Sequence Tables For Multiples??

- Students become experts at writing "general rules."
- These general rules help students describe a rule for finding the n th term in a sequence
- What they are really writing are algebraic expression and equations!



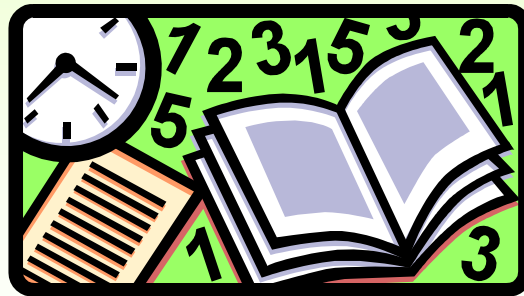
Algebra in Grade 7



By
Jennifer Quadrini

Algebra in Grade 7

In seventh grade, students are exposed to algebra in the first lesson. Students work with a variety of different problems including patterns, solving for known and unknown variables, writing equations in both words and symbols



Algebra in Grade 7

Write the equation that represents 8 more than the term number

If you wrote...

$$8 + n = t$$

You are correct!!!



Algebra in Grade 7

What is the value of X?

If...

$$3 + X = 8$$

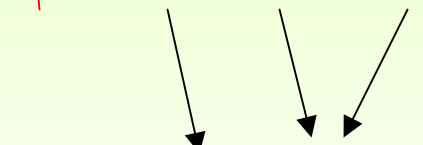
What is the value of
x?

To solve $3 + X = 8$

Find the inverse of 3

$$3 + X = 8$$

$$-3 + X = 8 - 3$$


$$X = 5$$